Mathematical Modeling: An Important Tool for Mathematics Teaching

[1]Ashim Bora, [2]Sahin Ahmed

[1]Associate Professor and Head, [2]Professor and Head
[1, 2] Department of Mathematics
[1]Diphu Government College, Diphu, Assam, India
[2]Rajiv Gandhi University, Rono Hills. Arunachal Pradesh, India

Abstract : The motivation behind this examination is to present the theoretical structure of Modeling Activities, which is believed to be a vital device for mathematics instruction. Modeling activities are defined as activities to figure out complex problems faced in real life situations that require the creation of a mathematical model as a product. In order to introduce the modeling activities within the scope of the study, the process of the developments of these activities is given in historical sequences and how they are defined by different educationists in the literature. Different steps of mathematical model formation, principles of mathematical model are illustrated. The importance of modeling activities in mathematics teaching, its different components and how they should be applied in courses are also included.

IndexTerms - Mathematical Modeling, Modeling Principles and Processes, Model Eliciting Activity, Mathematics Teaching.

I. INTRODUCTION

A Mathematical representation is a depiction of a framework utilizing Mathematical ideas and language. The route toward building up a Mathematical model is named as Mathematical Modeling. Specialized researchers have shown mathematical modeling (MM) in various ways. Mathematical modeling can be depicted as the procedure of giving information about any issue, understanding, painting, and giving light out of mathematics. Lesh and Lehrer (2003) marked MM a movement and a movement to find quantitative examples of its speculation. As per the Encyclopedia Britannica a "*mathematical model is either a physical representation of mathematical concepts or mathematical representation of reality*". In Wikipedia.org, the definition of mathematical model is found as:

"*A mathematical model is a description of a system using mathematical concepts and language. The process of developing a mathematical model is termed mathematical modeling. Mathematical models are used in the natural sciences and engineering disciplines as in the social sciences".*

Blum & *Leiβ* (2007) endeavor to clear up the fundamental ideas and terms identified with the term mathematical model and the displaying procedure. Lesh & Doerr (2003) described mathematical models are applied frameworks comprising of components, relations, tasks, and decides overseeing communications that are communicated utilizing outer documentation framework, and that are utilized to build, depict, or clarify the practices of different frameworks maybe with the goal that the other framework can be controlled or anticipated wisely. Yoon, Dreyfus & Thomas (2010) stated that mathematical modeling task begins with a genuine issue and is done by mathematizing the issue and discovering its answer and deciphering this arrangement in regards to reality. Real life problems are the starting points for modeling activities, and these activities are seen as an ideal way to identify and understand the aspects of mathematics in real life, learning mathematics and real life.

A mathematical model spotlights on basic attributes of the application frameworks. Mathematical modeling (MM) can be depicted as a roundabout and multidimensional problem-solving process that includes making an interpretation of genuine issues into mathematical language, illuminating those utilizing scientific procedures and in this manner testing the arrangements (Blum & Niss 1991; Haines & Crouch, 2007). Barbosa (2012) stated that MM is a learning domain where learners are welcome to ask and research, by methods for mathematics, circumstances emerging in different territories of learning. In case we take a method of socio-critical, the inquiry goes past the definition or comprehension of an issue, coordinating the learning of mathematics, modeling and thoughtful. Kaiser & Sriraman (2006) expressed that there are various ways to deal with the utilization of MM in mathematics teaching, and a typical methodology presently can't seem to be built up. These methodologies can be characterized in two classifications by the reason for their utilization in mathematics instructions. Firstly MM as the goal of mathematics guidance, and secondly, MM as an apparatus utilized in mathematics teaching. As indicated by the first methodology, MM is the capacity to utilize the dynamic ideas educated in mathematics, all things considered, and a fundamental expertise that learners ought to obtain (Lingefjard & Holmquist, 2005). An example of later approach uses the term 'model-eliciting activities'(MEA) for modeling problems, and emphasizes the use of these activities in mathematical instructions. The second approach called MEA proposes that altogether six standards ought to be contemplated in the development of exercises. The present study embraced the second methodology MEA and concentrated on the reality standard among these six standards.

In order to engage children in mathematical modeling, a class of issues that actually illustrate complex models as a model-eliciting activity can be utilized to deal with the need to develop models by expressing, examining and refining their mathematical ideas (Eric, 2008). During the MM process it can be observed as an internal representation (e.g. a table or a graph), while also in the form of internal ideological representation. These portrayal frameworks can be viewed as covering associated and intelligent. Including all these properties, Eric (2008) has given a diagrammatic portrayal of the modeling process (Fig. 01).

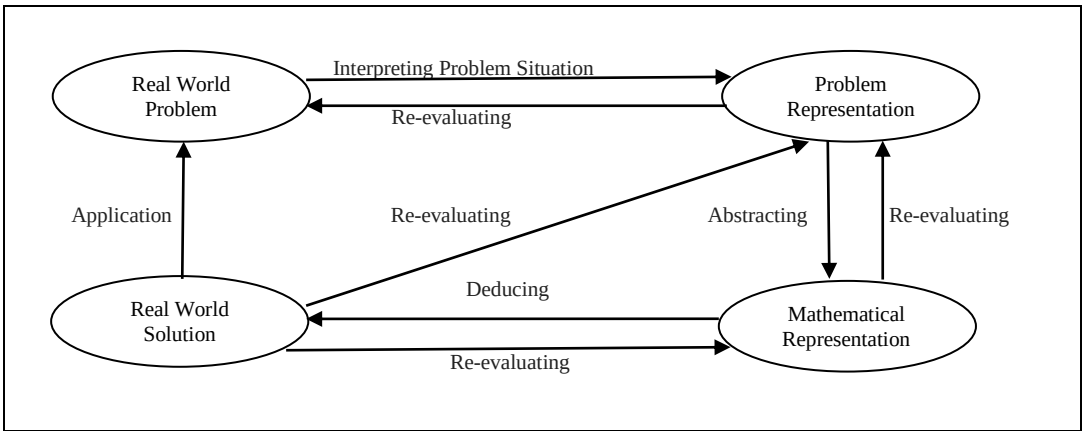

Fig.01: A general mathematical modeling process (Eric, 2008)

Research has indicated that in order to facilitate mathematical modeling activities for teachers in mathematics, they should be familiar with the mathematical modeling process (Eric, Dawn, Wanty, Seto, 2015).

II. Relevance of Mathematical Modeling

Mathematical modeling is seen as a fundamental capability, and the purpose of educating mathematics is to equip learners with this ability to take care of real life issues in mathematics and other realm (Blomhøj & Jensen, 2007; Blum, 2002; Lingefjard & Holmquist, 2005). MM is the procedure of critical thinking by a real life event or mathematical expression of a problem and this process enables learners to connect mathematics to real life and to learn it more meaningful and permanent (Arseven, 2015). Blum & Niss (1991) discussed five main arguments regarding inclusion of mathematical modeling in mathematics courses. Those arguments have been presented in detail.

Formative argument:

MM in mathematics teaching help students to develop common abilities, perspectives, and self-confidence by promoting the ability to solve the overall problem that is inventive, creative and open minded. Teaching should give learners the chance to challenge, develop and widen their inventiveness and logical aptitudes. Mathematics teaching ought to reinforce learners' trust in their capacity to utilize mathematics in various settings, and give degree to critical thinking both as an objective and an instrument.

Critical competence argument:

Mathematical Modeling in mathematics education help to get ready students to be incredulous of mathematics utilized in private life and in the public sphere, which means to have the capacity to freely recognize, break down and comprehend circumstances and examples where mathematics is being utilized. Teaching ought to fortify learners' trust in their capacity to comprehend its significance for the individual and society.

Utility argument:

MM in mathematics education teaches that by the utilization of mathematics instructions make learners mindful of how mathematics can be used in various circumstances particularly identified with the additional numerical area. Instruction should happen in conditions that are pertinent and firmly identified with learning in a practical way.

Picture of mathematics argument:

Mathematics Modeling might be used to give the learners an expansive and bright picture of mathematics as a science, as a movement in the public eye and in culture. Mathematics education should provide students experience in logic, generality, creative qualities and versatile nature of mathematics.

Promoting mathematics learning argument:

Mathematical modeling in mathematics education helps to assist and motivate learners to acquire mathematical concepts and methods.

III. Differences between Model-Eliciting Activities and Traditional Word Problem:

This segment will analyze traditional problem solving exercises and MEAs in their specific situations. Many researchers (Erbaş, Çetinkaya, Alacacı, Kertil, Çakıroğlu, and Baş, 2014) outlined the utilization of MEAs in mathematics teaching as vital. When a subject is done before or after the direct instruction, then these activities do to develop their own understanding of the students or to deepen their conceptual comprehension of the department (Lesh et al., 2007; Yoon et al., 2010). Traditionally, it is believed that problems of real life are more difficult to solve than traditional problems of the curriculum, and after getting some progress in troubleshooting instructions, the issues can be answered. The real life issues are alluded to as connected critical thinking in the conventional critical thinking approach, and they are viewed as a particular type of customary critical thinking. Lesh and Doerr demonstrated that it is less demanding to handle important real life issues and that they give further and more significant mathematical learning (Lesh and Doerr, 2003). In the primary and middle school mathematics course, it is important that students can develop effective solutions for problems faced during the primary purpose of enhancing successful people in life. Contrary to traditional oral problems in the textbooks, students develop mathematical models, discuss their approach with the group's partner during the solution process, and develop mathematical models that benefit from the mathematically optimized real-life situation. Model-eliciting ventures are more realistic than conventional term problem. The table 01 clearly reflects the truth that Model-eliciting ventures are more realistic than traditional word problems solving activities.

Model Eliciting Activities	Traditional Problem Solving Activities
It contains genuine components and utilizations them for the arrangement.	It might have settings that are unrealistic in real life.
The issue setting is consistent with life.	Are not viewed as issues, in actuality.
The setting of the action helps us to remember our very own encounters.	Problems are made by the situation of a mathematical structure.
It is a problem that anyone can cope in real life more often.	It's time to end up being traditional.
Instructors and learners assemble the arrangement themselves.	Problems are constructed with the aim of composing an inquiry.
Teachers and students use their real-life encounters to create a problem.	Students solve the problems just with a motive of solving them.
The solutions found can be used in reality.	Students resolve to be ready for examinations, ignoring real life references.
The answers to modeling issues are really important.	Solutions may not be useful in real life status.
Problems are for everyone and are reusable.	Problems and their answers are suitable for special circumstances only.

TABLE: 01 Model eliciting activities verses Traditional Problem Solving Activities

IV. Aim and Objective of the Study:

The reason behind this examination is to present the hypothetical structure of model inspiring exercises considered to a vital instrument for mathematics training, to embody a model evoking movement built by mathematics educators and to clarify its application procedure. In a normal way, model development activities are defined as essential real-life troubleshooting activities to create a mathematical model. Modeling- Eliciting Activities (MEA) characterizing non-routine complex true circumstances are not well characterized and open-finished issues expecting students to reason and decipher about the circumstance, and to characterize and formalize the procedure mathematically to help clients who profit by this circumstance. The aim and objective of this examination is to present theoretical structure of modeling activities, which is considered to be a significant tool for mathematical instructions.

V. Principle of Model-Elcit Activities:

Research in the field of MM records six standards for creating mathematical modeling exercises. To create mathematical modeling exercises, designers depend upon these six rules that depend on the exercises of educator and students and that have after refined by Lesh & Doerr (1998). The principles are Model Construction, Reality, Self Assessment, Documentation, Sharing and Re-Usability, Effective Prototype.

The primary guideline for designing mathematical modeling endeavors is called '*Model Construction Principle*'. This standard guarantees that the answer for the contextual investigation requires the development of an express depiction, clarification, method, or legitimized forecast for a given numerically huge circumstance. Such items externalize how the students translate the circumstance and furthermore reveal the sorts of mathematical scores, connections, tasks, and example that they consider. The issue circumstance expects learners to make a mathematical model as a substitute for creating a word or a number as an item.

The second plan rule is the '*Reality Principle*'. This rule could likewise be alluded to as the seriousness rule. It identifies with two vital qualities of a contextual analysis. To begin with, it requires the contextual analysis to be composed so the learners can decipher the movement definitively from their diverse levels of mathematical capacity and general learning. Besides, this guideline requires the demonstrating action to represent an issue that could occur in real life situations. The substance of mathematical modeling ought to incorporate circumstances that can be significant to the learners' genuine lives, and learners ought to make a model for a genuine individual who is looking for assistance from them.

The third plan standard is *'Self Assessment Principle'*. This rule ensures that the modeling activity contains criteria used to identify students and examine and correct their current ideas. In particular, the modeling activity should include information that students can use to evaluate the usability of their alternative solutions, to determine when and how to improve their solutions, and when they are finished. In the event of problem, the students should be required to decide on the degree to which they have been able to make decisions by discussing solutions developed with their group's friends.

The fourth standard is *'Documentation Principle'*. It guarantees that while demonstrating movement, the learners are required to make some type of documentation that will uncover expressly how they are considering the issue circumstance. Requiring outside documentation of their reasoning is advantageous for both the learners and educators. The documentation is useful for the instructors since it uncovers how the students are translating about the give circumstance. Besides, the documentation is helpful for the learners since when learners externalize their reasoning; it winds up less demanding for them to self access or to ponder their reasoning. After all, externalizing their reasoning enables learners to participate in self-understanding. This guideline is commonly refined in two distinct ways. The students work in little gatherings so that they expressly uncover their reasoning when they speak with one another to do process, for example, arranging, observing, and evaluating their answers. Besides, the issue is begun to expect learners to deliver clarification, strategy, depiction as a feature of their answers. These two procedures deliver documentations that uncover how learners are considering the given circumstance.

The fifth guideline is the *'Sharing and Re-Usability Principle'* which expects learners to create a share-capable and re-usable arrangement. By requesting that the students deliver an item that can be utilized by others beyond the quick arrangement, modeling affairs demand the learners go beyond individual methods of thinking to originate extra broad paths of thoughts, ensuring in more strong estimations. This rule is also known as Model Generalization Principle. According to this principle, it is important for students not only to be used for a specific situation and for a specific purpose, but also to be participated with others and to develop reusable models in parallel situations for different purposes. Chamberlin and Moon (2005) express that the arrangement can be regarded as effective if the created model is summed up to various circumstances that require a comparable model.

The last and the sixth rule is *'Powerful Prototype Principle'* which guarantees that the demonstrating action will be as straightforward as conceivable yet still mathematically consequential. The objective is for learners to create arrangements that will give models to translating other comparable circumstances. It is seen that the guideline of effective prototype model is like the model generalization standard (Dede , Hıdıroğlu & Güzel, 2017). The presence of the difference is clarified by Chamberlin and Moon (2005) and expressed that learners should utilize the model they prepared in a similar but not parallel circumstance.

VI. Advantages and Disadvantages of Mathematical Modeling Process:

The points of interest and inconveniences of mathematical modeling activities in the classroom can be condensed as follows.

Advantages:

i) The learners are more inspired by an action, for example, scientific demonstrating than taking in the specific circumstance, taking care of a few issues, and figure out how to explain a condition, without knowing how the issue can be connected in real life world, since when all is said in done, the mathematics issues have no significance for learners, not notwithstanding for educators.
ii) The learning has a genuine importance. For learners as well as teachers, it becomes easy to establish connections to other situations and problems.
iii) Students learn approaches to assemble associations with other situations, especially physical situations. Students feel more ready and confident to use mathematics in different areas of studies.
iv) It can be applied in any level of school education, primary and middle education curriculum.
v) Mathematical modeling process is more adaptable and obedient for an instructor than the orthodox mathematics learning methods.

Disadvantages:

i) The learners normally dislike adopting a new procedure to tackle a problem.
ii) The choosing of good problem to discuss in the classroom is not very simple for learners, so normally, and actually is the art of the instructor.
iii) The MM activities require additional time than the conventional learning techniques.
iv) It is hard to comprehend the setting encompassing a given circumstance.
v) It is hard to recognize the basic parts of a set circumstance and make an interpretation of them into mathematical terms

VII. Obstacles for Mathematical Modeling:

In this section some obstacles for MM in the classroom are discussed. In many nations, scientific mathematical modeling still assumes just a minor job in regular classroom practice at all levels of formal education from elementary to university. Formal numerical calculations are broadly used, and application precedents are dealt with just as representations and are not considered important in that capacity. Indeed, even capable and submitted instructors frequently don't pursue every one of those very much established proposals and recommendations of mathematics education teachers. This may happen not due to "ill-will or incompetence" but due to few actual barriers (Blum, 1993). These barriers may be summarized as:

Teaching and Estimation: Mathematical modeling requires a considerable measure of time and fits neither the standard mathematics syllabus which is over-burden in any case, nor the consistent school or college association. Mathematical modeling is hard to evaluate as mathematical modeling process must travel through six principles, and what isn't inspected won't be considered important by learners or by instructors.

Students' Approaches: More engagement of students is demanded by this learning approach. Mathematical modeling makes mathematics exercises and examinations all the more demanding and consequently more uncertainty for learners.

Teachers' Approaches: Mathematical modeling makes encouraging all the more requesting on the grounds that extra non-scientific information and capabilities are required, and also the capacity to deal with an open circumstance in the classroom. So the educator's job is changed. Moreover, instructors frequently don't know enough mathematical models reasonable for guidance, and the ones they have close by must be raised to-date and must be adjusted for the class, for which a lot of extra exertion must be contributed. There are insufficient mathematical modeling precedents and materials reasonable for teaching.

VIII. Conclusion:

In this examination we have exhibited the speculative structure of model motivating activities considered to an essential instrument for mathematics teaching, to epitomize a model summoning development worked by mathematics education educators. We tried to clear up its application of mathematical modeling approach as an alternative strategy in the mathematics education. The mathematical modeling, truth be told, attempts to interface learners to an assortment of thoughts and abilities, and in this instruction procedure, learners can figure out how to make an association with different circumstances feel more arranged to the utilization of mathematical knowledge in different zones. From our own involvement in teaching mathematics, we can tell that the mathematical modeling approach, as another option in learning, rolls out astounding improvements accessible to students and educators, and these progressions together recommend that the mathematical modeling enhances the learning procedure since the learners take in more by observing the things and secure it by mirroring it over and over.

References

[1] Arseven, A. (2015). Mathematical Modelling Approach in Mathematics Education. Universal Journal of Educational Research. 3(12), 2015, 973-980.
[2] Barbosa, J. (2012). Mathematical modeling, mathematical content and tensions in discourses. In G. A. Stillman, G. Kaiser, W. Blum, & J. P. Brown (Eds.), Teaching mathematical modeling: Connecting to research and practice ,Dordrecht, The Netherlands: Springer, 67-76.
[3] Blomhøj, M., & Jensen, T. H.(2007). What's all the fuss about competencies? In W. Blum, P. L. Galbraith, H. Henn, & M. Niss (Eds.), Modelling and applications in mathematics education. The 14th ICMI study, New York, NY: Springer, 45-56.
[4] Blum, W. (1993). Teaching and learning mathematics in context, Edited by Breiteig (etc.), Ellis Horwood Limited, 3-14.
[5] Blum, W. (2002). ICMI Study 14: Applications and modelling in mathematics education-Discussion document. Educational Studies in Mathematics, 51(1-2), 49-171.

[6] Blum, W., & Leiβ, D. (2007). How do students and teachers deal with modelling problems? In C. Haines, P. Galbraith, W. Blum, & S. Khan (Eds.), Mathematical modelling (ICTMA 12): Education, engineering and economics: proceedings from the twelfth International Conference on the Teaching of Mathematical Modelling and Applications, Chichester: Horwood, 222-231.

[7] Blum, W., & Niss, M.(1991). Applied mathematical problem solving, modelling, applications, and links to other subjects – state, trends and issues in mathematics instruction. Educational Studies in Mathematics, 22(1), 37–68.

[8] Chamberlin, S. A. and Moon, S. M. (2005). Model eliciting activities as a tool to develop and identify creatively gifted mathematicians. The journal of secondary gifted education. 179(1), 37-47.

[9] Dede, A.T., Hıdıroğlu, Ç.N., & Güzel, E.B. (2017). Examining of Model Eliciting Activities Developed by Mathematics Student Teachers. Journal on Mathematics Education, 8(2), 223-242.

[10] Erbaş, A. K., Kertil, M., Çetinkaya, B., Çakıroğlu, E., Alacacı, C., & Baş, S. (2014). Mathematical modeling in Mathematics education: Basic concepts and different approaches. Educational Sciences: Basic Concepts and Different Approaches Practice, 14(4), 1607–1627.

[11] Eric, C M., Dawn, N K., Wanty,W. & Seto, C.(2015). A Case Study on Developing a Teacher's Capacity in Mathematical Modelling. The Mathematics Educator 16(1), 45-74.

[12] Eric, M. (2008). Using Model-Eliciting Activities for Primary Mathematics Classrooms. The Mathematics Educator, 11(1), 47-66.

[13] Haines, C., & Crouch, R.(2007).Mathematical modelling and applications: Ability and competence frameworks. In W. Blum, P. L. Galbraith, H. Henn, & M. Niss (Eds.), Modelling and applications in mathematics education: The 14th ICMI study, New York, NY: Springer, 417-424.

[14] Kaiser, G & Sriraman, B.(2006). A global survey of international perspectives on modelling in mathematics education. ZDM Mathematics Education, 38(3), 302-310.

[15] Lesh, R., & Doerr, H. M. (2003). Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching (Mahwah, NJ: Lawrence Erlbaum Associates.

[16] Lesh, R., & Lehrer, R. (2003). Models and Modeling Perspectives on the Development of Students and Teachers. Mathematical Thinking And Learning, 5(2&3), 109–129.

[17] Lingefjärd, T., & Holmquist, M.(2005). To Assess Students' Attitudes, Skills and Competencies in Mathematical Modeling. Teaching Mathematics and its Applications, 24(2-3), 123-133.

[18] Lingefjärd, T., & Holmquist, M. To assess students' attitudes, skills and competencies in mathematical modeling. Teaching Mathematics and Its Applications, 24(2- 3), 2005, 123-133.

[19] Lesh, R., Yoon, C., & Zawojewski, J. (2007). John Dewey revisited: Making mathematics practical versus making practice mathematical. In R. A. Lesh, E. Hamilton, & J. J. Kaput (Eds.), Foundations for the future in mathematics education. Mahwah, NJ: Lawrence Erlbaum Associates, 315-348.

[20] Lesh, R., & Doerr, H. M. (2003). Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching, Mahwah, NJ: Lawrence Erlbaum Associates.

[21] Lesh, R., & Doerr, H.M.(1998). Symbolizing, communicating and mathematising: Key concepts of models and modeling. In P. Cobb, E. Yackel, & K. McClain (Eds.), Symbolizing and communicating in mathematics classrooms. Mahwah: Lawrence Erlbaum Associates, 1998, 361−394.

[22] Yoon, C., Dreyfus, T., & Thomas, M. O.(2010). How high is the tramping track? Mathematising and applying in a calculus model-eliciting activity. Mathematics Education Research Journal, 22(2), 141-157.